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## THE OLD AND THE NEW OF PARITY-VIOLATING TWO-PION-EXCHANGE $NN$ POTENTIAL

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We consider the parity-violating two-pion-exchange potential obtained from the covariant formalism in the past and the state-of-the-art effective field theory approach. We discuss the behavior of the potential in coordinate space and its application to the parity-violating asymmetry in  $\bar{n}p \rightarrow d\gamma$  at threshold.

*Keywords:* Nucleon-nucleon interaction; Applications of electroweak models to specific processes

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### 1. Introduction

Effective field theory (EFT) approach, which shed new light on the methodology of nuclear physics, was used to reformulate the parity-violating (PV) nucleon-nucleon ( $NN$ ) interaction recently,<sup>1</sup> and the potential has been applied to a few two-nucleon problems.<sup>2,3</sup> The PV two-pion-exchange (TPE) contribution to physical observables turned out to be non-negligible, but was not so significant as the TPE in the strong  $NN$  interaction. On the other hand, a PV  $NN$  contact term parametrized by a low-energy constant (LEC), which is presumed to subsume the heavy degrees of freedom and higher-order corrections, is comparable to the PV TPE contribution.

More importantly, the LEC term (the contact term) is highly singular at short distances and depends critically on the regularization methods and renormalization schemes. With the lack of experimental data to determine the LEC value, it is not feasible to sharpen the theoretical prediction furthermore.

As a partial resolution to the uncertainty associated with the LEC term, we consider the covariant formulation of the PV TPE potential, which was already carried out in 1970's.<sup>4,5,6</sup> Compared to the TPE and LEC terms in EFT, the covariant formalism gives a potential which is finite and has no unknown constant. Moreover, it is less singular at short distances than that from EFT, and contains more higher-order  $1/M$  contributions. In this work we focus on comparing the TPE potential from the covariant formalism with that from EFT. By calculating a PV asymmetry in  $\vec{n}p \rightarrow d\gamma$  with these potentials, we estimate the correction from the higher-order terms embedded in the covariant potential. The analysis will provide an indirect estimation of the correction due to the LEC term in EFT.

## 2. Formalism

The PV TPE potential contains various spin and isospin components. Keeping the spin and isospin operators only relevant for the asymmetry  $A_\gamma$  in  $\vec{n}p \rightarrow d\gamma$ , we get the following expression :

$$V_{\text{TPE}}^{\vec{n}p}(\vec{r}) = i(\vec{\tau}_1 \times \vec{\tau}_2)^z (\vec{\sigma}_1 + \vec{\sigma}_2) \cdot [\vec{p}, v_{56}(r)] + (\tau_1^z - \tau_2^z) (\vec{\sigma}_1 + \vec{\sigma}_2) \cdot \{\vec{p}, v_{75}(r)\}. \quad (1)$$

The radial part of the potential in the covariant approach can be written in the form

$$v_{ij}^{\text{cov}}(r) = \frac{1}{4\pi^2} \int_{4m_\pi^2}^{\infty} dt' g_{ij}(t') \frac{e^{-r\sqrt{t'}}}{r\sqrt{t'}}, \quad (2)$$

where  $m_\pi$  is the pion mass and  $g_{ij}(t')$ 's are the spectral functions for given indices  $i$  and  $j$ . We have  $g_{56}$  and  $g_{75}$  as

$$\begin{aligned} g_{56}(t') &= \frac{g_{\pi NN}^3 h_\pi^1}{32\sqrt{2}\pi} \left[ -\frac{1}{2M} \frac{Hx}{x^2 + 4M^2 q_\pi^2} - \frac{x}{M^2 m_\pi^2} \text{arctg}\left(\frac{m_\pi^2}{2Mq_\pi}\right) \right. \\ &\quad \left. + \int_{k_-^2}^{k_+^2} \frac{dk^2}{\sqrt{k^2 t' - (m_\pi^2 + k^2)^2}} \frac{1}{2E(E+M)} \left( \frac{x}{M^2} + \frac{k^2}{M(E+M)} \right) \right], \\ g_{75}(t') &= \frac{g_{\pi NN}^3 h_\pi^1}{32\sqrt{2}\pi} \left[ \frac{4x^2}{M^2 m_\pi^2 t'} \text{arctg}\left(\frac{m_\pi^2}{2Mq_\pi}\right) + \frac{2G}{Mt'} \right. \\ &\quad \left. - \int_{k_-^2}^{k_+^2} \frac{dk^2}{\sqrt{k^2 t' - (m_\pi^2 + k^2)^2}} \frac{2}{E(E+M)} \left( \frac{x^2 + 2EMx}{M^2 t'} - \frac{k^2 t' + k^4}{M(E+M)t'} \right) \right], \quad (3) \end{aligned}$$

where  $M$  is the nucleon mass and  $h_\pi^1$  the weak  $\pi NN$  coupling constant. The definitions of the functions  $q_\pi$ ,  $\chi^2$ ,  $x$ ,  $H$ ,  $G$ ,  $k_\pm^2$  and  $E$ , which are non-linear functions of  $t'$  and  $M$ , can be found in Ref. 7. Due to the non-linearity of these functions, when expanded in terms of  $1/M$ ,  $g_{56}$  and  $g_{75}$  have infinite terms in the power of  $1/M$ .

In order to understand the relation between the covariant and EFT approaches, we take the large nucleon mass (LM) limit for the spectral functions. Keeping only the leading terms of  $1/M$  in each function, we have

$$\lim_{M \rightarrow \infty} g_{56}(t') = -\frac{g_{\pi NN}^3 h_\pi^1}{32\sqrt{2}\pi} \frac{x}{q_\pi M^3}, \quad \lim_{M \rightarrow \infty} g_{75}(t') = \frac{g_{\pi NN}^3 h_\pi^1}{32\sqrt{2}\pi} \frac{\pi}{4M^4} (q_\pi^2 + 3x). \quad (4)$$

For  $g_{56}$  the leading order is  $1/M^3$ ; for  $g_{75}$ , it is  $1/M^4$ . Consequently,  $v_{56}$  becomes the leading term in LM, and  $v_{75}$  is higher-order in  $1/M$ . We have explicit form for  $v_{56}$  in momentum space as

$$v_{56}^{\text{LM}}(q) = -\sqrt{2}\pi \frac{g_{\pi NN}^3 h_\pi^1}{(4\pi g_A M)^3} \left[ g_A^3 L(q) - g_A^3 \left( 3 - \frac{4m_\pi^2}{4m_\pi^2 + q^2} \right) L(q) \right], \quad (5)$$

where  $g_A$  is the axial coupling constant and the definition of  $L(q)$  can be found in Ref. 2. The component  $v_{56}^{\text{LM}}$  given by Eq. (5) is almost the same as the corresponding TPE term in EFT, except for the factor  $g_A^3$  of the first term in the square bracket, which is  $g_A$  in EFT.<sup>2</sup> The discrepancy is explained in Ref. 7, and it is concluded that the leading  $1/M$  term extracted from the covariant form is in principle equivalent to the TPE term obtained from EFT. In the next section, we present numerical results for the potentials and their application.

### 3. Numerical Results

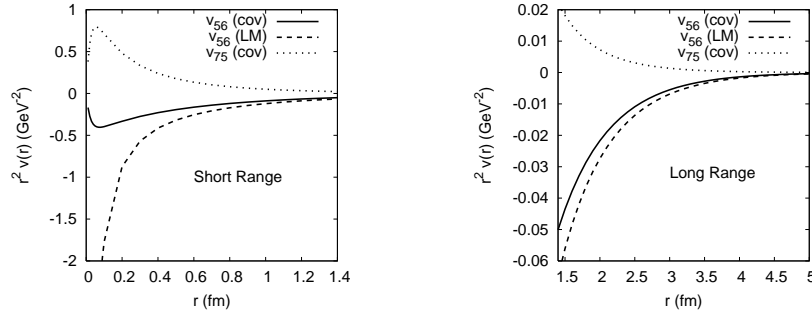


Fig. 1. TPE potentials in the short (left) and the long (right) range region.

Fig. 1 shows the  $v_{56}$  and  $v_{75}$  components of the PV TPE potential in coordinate space multiplied by  $r^2$ . In the short-range region (left panel), the covariant terms  $v_{56}$  and  $v_{75}$  both show non-singular behavior around  $r = 0$ , while the LM term diverges as  $r \rightarrow 0$ . We expect that when a properly renormalized LEC term is added to  $v_{56}^{\text{LM}}$ , it will make the net behavior less singular at small  $r$ . On the other hand, the covariant and LM forms show convergent behavior in the long-range region (right panel).

The PV asymmetry  $A_\gamma$  in  $\vec{n}p \rightarrow d\gamma$  is defined as the photon asymmetry with respect to the neutron polarization. Treating  $h_\pi^1$  perturbatively and keeping only the

Table 1. Numerical results for the asymmetry.  $v_{\text{OPE}}$  represents the PV one-pion-exchange (OPE) potential.

Potential	$v_{\text{OPE}}$	$v_{56}^{\text{cov}}$	$v_{75}^{\text{cov}}$	$v_{56}^{\text{LM}}$
$a_\gamma$	-0.1120	0.0093	-0.0040	0.0141

leading-order contribution,  $A_\gamma$  is proportional to  $h_\pi^1$ , i.e.  $A_\gamma = a_\gamma h_\pi^1$ . The factor  $a_\gamma$  depends on both strong and weak dynamics. In order to reduce the uncertainty from the strong interaction, we employ the Argonne v18 potential to obtain the parity-conserving part of the wave function. Numerical results for  $a_\gamma$  are summarized in Table 1. The covariant contributions from  $v_{56}$  and  $v_{75}$  are summed to cancel strongly,  $0.0093 - 0.0040 = 0.0053$ , which is much smaller than the one-pion-exchange (OPE) one,  $-0.1120$ . Comparing the contribution of  $v_{56}^{\text{cov}}$ , which contains infinite orders of  $1/M$ , with  $v_{56}^{\text{LM}}$ , which is proportional to  $1/M^3$ , the higher-order  $1/M$  corrections are non-negligible, but less significant than the leading  $1/M$  contribution.

In summary, the behavior of the potential at short distances and the PV asymmetry in  $\vec{n}p \rightarrow d\gamma$  show that the uncertainty due to the corrections from the degrees of freedom integrated out of the model is still significant, and it requires further investigation. However, the TPE and higher-order contributions to  $A_\gamma$ , in either covariant or EFT form, gives a correction about 10 % at most. Recent calculations of the  $\Delta(1232)$  resonance contribution to the PV potential in EFT show that the present result of  $A_\gamma$  is not much affected by the  $\Delta$  contribution.<sup>8</sup> To the extent investigated so far,  $A_\gamma$  is dominated by the PV OPE potential with an uncertainty of about 10% at most. The precise measurement of  $A_\gamma$  will thus provide a unique opportunity to determine the value of  $h_\pi^1$ .

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